THE NUMERICAL SOLUTION OF RADIATIVE HEAT TRANSFER FOR GREY BODIES IN AN ABSORBING MEDIUM

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A new approximate analytical method for solving the integral radiation equations [1, 2] is used for the numerical calculation and investigation of the local and average characteristics of radiative heat transfer in systems of grey bodies separated by an isothermal absorbing medium.

The Solution of the Mixed Two-Dimensional Problem of Radiative Heat Transfer in a Chamber of Rectangular Cross Section. A chamber of rectangular cross section of infinite length consists of three optically homogeneous bounding grey bodies (one at each end with degrees of blackness A_1, A_3, and an adiabatic one in the middle with surface $Q_{res,2} = E_{res,2}F_2 = 0$) separated by an isothermal absorbing medium with given temperature T_4 and coefficient of volume absorption α. In addition the geometrical dimensions of the chamber and the temperatures of the ends T_1 and T_3 are defined.

It is required to determine the field of values of the surface density of the resulting radiation from the ends $E_{res}(M_i)M_i \in F_i$ $(i = 1,3)$ and the temperature field $T(M_2)$ of the lateral surface F_2 of the chamber.

The fundamental computational equations and expressions in the most general nondimensional form in this case are [3]:

$$\theta_{res}(M_1) = \frac{E_{res}(M_1)}{E_{41}} = A_1\left[\mathfrak{A}(M_1, V) + A_3\theta_{31}\Psi(M_1, F_3)\right], \tag{1}$$

$$\theta_{res}(M_3) = \frac{E_{res}(M_3)}{E_{41}} = A_3\left\{\mathfrak{A}(M_3, V) - \left[1 - A_3\Psi(M_3, F_3)\right]\theta_{31}\right\}, \tag{2}$$

$$\theta(M_2) = \frac{T^4(M_2) - T_4^4}{T_1^4 - T_4^4} = 1 - \mathfrak{A}(M_2, V) - A_3\theta_{31}\Psi(M_2, F_3), \tag{3}$$

where $\theta_{31} = (T_3^4 - T_1^4)/(T_4^4 - T_1^4)$.

In Eqs. (1)-(3) $\mathfrak{A}(M_i, V)$ is the local resolving absorptivity of the medium, [4]

$$\mathfrak{A}(M_i, V) = 1 - A_1\Psi(M_i, F_1) - A_3\Psi(M_i, F_3) \quad (M_i \in F_i, \quad i = 1, 2, 3). \tag{4}$$

If in (4) we make the subscript i take the values 1, 2, 3, in turn we obtain

$$\mathfrak{A}(M_1, V) = 1 - A_1\Psi(M_1, F_1) - A_3\Psi(M_1, F_3) \quad (M_1 \in F_1), \tag{5}$$

$$\mathfrak{A}(M_2, V) = 1 - A_1\Psi(M_2, F_1) - A_3\Psi(M_2, F_3) \quad (M_2 \in F_2), \tag{6}$$

$$\mathfrak{A}(M_3, V) = 1 - A_1\Psi(M_3, V_1) - A_3\Psi(M_3, F_3) \quad (M_3 \in F_3). \tag{7}$$

Expressions for the average resolving absorptivities of the media $\mathfrak{A}_i(V)$ $(i = 1, 2, 3)$ can easily be obtained from expressions (4)-(7) for $\mathfrak{A}(M_i, V)$ by replacing in them the local resolving angular radiation coefficients $\Psi(M_i, F_n)$ by the corresponding average resolving angular coefficients Ψ_{in}. In turn replacing the functions $\mathfrak{A}(M_i, V)$, $\Psi(M_i, F_n)$ by the corresponding $\mathfrak{A}_i(V)$, Ψ_{in} in Eqs. (1)-(3) makes it possible to obtain computational expressions for the average nondimensional boundary radiation characteristics $\theta_{res,1}$, $\theta_{res,3}$, and θ_2.

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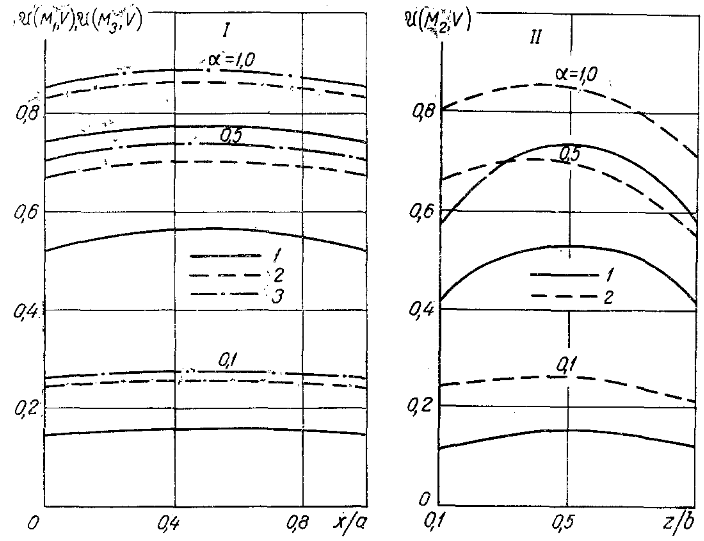

Fig. 1. Local resolving absorptivity of the medium as a function of x/a (I) and of z/b (II) for various values of α: I: 1) $\mathfrak{U}(M_1, V)$, $\mathfrak{U}(M_2, V)$, $A_1 = A_3 = 1.0$; 2)$-\mathfrak{U}(M_1, V)$; 3) $-\mathfrak{U}(M_3, V)$ [2 and 3) $A_1 = 0.3, A_3 = 0.7$]; II: 1)$-\mathfrak{U}(M_2, V)$, $A_1 = A_3 = 1.0$; 2) $A_1 = 0.3, A_3 = 0.7$.

For a radiating system which is a chamber of rectangular cross section with plain ends F_1 and F_3, the local and average generalized angular coefficients of self-radiation are zero ($\psi(M_1, F_1) = \psi(M_3, F_3) = \psi_{11} = \psi_{33} = 0$), and the coefficients of repeated reflection are $\gamma_1 = \gamma_3 = 1$. Hence the computational equations for determining the local resolving angular radiation coefficients obtained in [1] can be simplified and take the form:

$$D\Psi(M_1, F_1) = \psi_{21}'(1 + R_3\psi_{13})\psi(M_1, F_2) + R_3(\psi_{13} + \psi_{12}\psi_{21}')\psi(M_1, F_3), \tag{8}$$

$$D\Psi(M_2, F_1) = (1 - R_3\psi_{12}\psi_{21}')\psi(M_2, F_1)$$
$$+ \psi_{21}'(1 + R_3\psi_{13})\psi(M_2, F_2) + R_3(\psi_{13} + \psi_{12}\psi_{21}')\psi(M_2, F_3), \tag{9}$$

$$D\Psi(M_3, F_1) = (1 - R_3\psi_{12}\psi_{21}')\psi(M_3, F_1) + \psi_{21}'(1 + R_3\psi_{13})\psi(M_3, F_2), \tag{10}$$

$$D\Psi(M_1, F_3) = \psi_{21}'(1 + R_1\psi_{13})\psi(M_1, F_2) + (1 - R_1\psi_{12}\psi_{21}')\psi(M_1, F_3), \tag{11}$$

$$D\Psi(M_2, F_3) = R_1(\psi_{13} + \psi_{12}\psi_{21}')\psi(M_2, F_1)$$
$$+ \psi_{21}'(1 + R_1\psi_{13})\psi(M_2, F_2) + (1 - R_1\psi_{12}\psi_{21}')\psi(M_2, F_3), \tag{12}$$

$$D\Psi(M_3, F_3) = R_1(\psi_{13} + \psi_{12}\psi_{21}')\psi(M_3, F_1) + \psi_{21}'(1 + R_1\psi_{13})\psi(M_3, F_2), \tag{13}$$

$$D = 1 - R_1R_3\psi_{13}(\psi_{13} + 2\psi_{12}\psi_{21}') - \psi_{12}\psi_{21}'(R_1 + R_3), \tag{14}$$

where the R_k are the coefficients of reflection ($R_k = 1 - A_k$), $k = 1, 3$; $\psi_{21}' = \psi_{21}/(1 - \psi_{22})$ is the effective generalized average angular radiation coefficient from the lateral surface F_2 on the base F_1 of the chamber (a surface F_2 consists of two parallel unbounded strips).

From the symmetry of the radiating system (a chamber of rectangular cross section) it follows that $\psi(M_1, F_3) = \psi(M_3, F_1)$, $\psi(M_1, F_2) = \psi(M_3, F_2)$ for similar points M_1 and M_3 $\psi(M_2, F_1) = \psi(M_2, F_3)$ for the points M_2 symmetrically placed with respect to the axis of symmetry. For the average angular radiation coefficients we have respectively: $\psi_{13} = \psi_{31}$, $\psi_{21} = \psi_{23}$, $\psi_{32} = \psi_{12}$, $\psi_{22} \neq 0$.

The generalized local and average angular radiation coefficients for a chamber of rectangular cross section ($a/b = 0.5$) were determined using the approximate method of Mikk [5, 6] for various values of the coefficient of volume absorption ($\alpha = 0.1, 0.5, 1.0$). The local resolving angular radiation coefficients $\Psi(M_i, F_k)$ were computed from Eqs. (8)-(14) for the following values of the coefficients of reflection $R_1 = R_3 = 0$ and $R_1 = 0.7$, $R_3 = 0.3$.

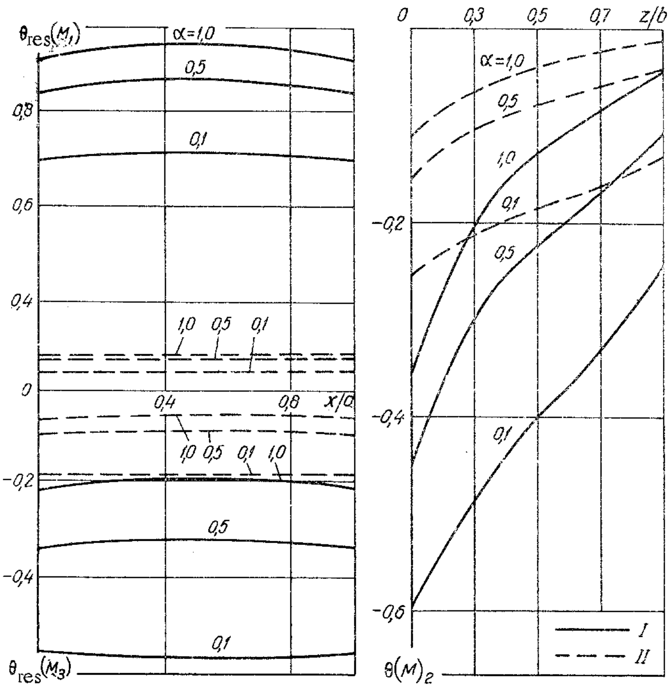

Fig. 2. The nondimensional density of the resulting radiation as a function of x/a (left) and the nondimensional temperature factor as a function of x/b (right) for various values of α: $\theta_{res,2} = 0$; $\theta_{31} = 1.05$; I) $A_1 = A_3 = 1.0$; II) $A_1 = 0.3$, $A_3 = 0.7$.

The results of the numerical calculations of the fundamental local characteristics of radiative heat transfer are shown in Figs. 1, 2. Figure 1 shows the graphical relations between the local resolving absorptivity of the media $\mathfrak{A}(M_i, V)$ and the nondimensional coordinates (x/a and z/b) of the base and lateral surface of the chamber for $a/b = 0.5$ and various values of A_1 and A_3 ($A_1 = A_3 = 1.0$ and $A_1 = 0.3$, $A_3 = 0.7$) and $\alpha = 0.1$, 0.5, 1.0. For $a/b = 0.5$ and fixed values of α and A_i ($i = 1, 3$) the local resolving absorptivity $\mathfrak{A}(M_i, V)$ of the volume V of the media depends weakly on the nondimensional coordinate x/a and increases both as the coefficient of volume absorption α increases and as the degree of blackness of the base (Fig. 1, I) decreases. For $A_1 = A_3$ the local resolving absorptivity of the medium $\mathfrak{A}(M_2, V)$ has its greatest value for the point $M_2 \in F_2$ with nondimensional coordinate $z/b = 0.5$. For $A_1 \neq A_3$ the maximum of $\mathfrak{A}(M_2, V)$ is displaced towards points on the lateral surface nearer the base with the lower degree of blackness (in this case towards the base F_1, since $R_1 > R_3$) (Fig. 1, II). Such a displacement is easily explained by the fact that the local resolving absorptivity $\mathfrak{A}(M_i, V)$ of the volume V of the medium, as distinct from the proper local absorptivity, takes into account the fact that there may be absorptions associated with repeated reflections at the boundary [4].

The results of the numerical calculations of the local energy characteristics of the radiation for given values of the parameters $a/b = 0.5$, $\theta_{res,2} = 0$, $A_1 = A_3 = 1.0$, $A_1 = 0.3$ and $A_3 = 0.7$, $\theta_{31} = 1.05$ for various values of the coefficient of volume absorption ($\alpha = 0.1$, 0.5, 1.0) are shown on Fig. 2. When α is constant the local nondimensional density of the hemispherical resulting radiation of the bases $\theta_{res}(M_1)$ and $\theta_{res}(M_3)$ changes little. For given $\theta_{31} = 1.05$, as the coefficient of volume absorption α increases $\theta_{res}(M_1)$ increases, while $\theta_{res}(M_3)$ decreases in absolute magnitude (Fig. 2, left). As the degree of blackness of the bases increases $\theta_{res}(M_1)$ and $\theta_{res}(M_3)$ increase in modulus.

The distribution of the nondimensional temperature factor $\theta(M_2)$ over the lateral surface (the lining) varies considerably (Fig. 2, right). The nondimensional temperature factor $\theta(M_2)$ decreases in the modulus both with increase in the coefficient of volume absorption α and with decrease in the degree of blackness of the bases.

The Solution of the Mixed Two-Dimensional Problem of Radiative Heat Transfer in a Radiating System Consisting of Two Concentric Grey Cylinders Divided by an Absorbing Medium. The radiating system consists of a pair of concentric grey ($0 < A_1, A_2 < 1$) infinite cylinders divided by a homogeneous and

TABLE 1. The Average Characteristics of the Radiation of Coaxial Infinite Cylinders ($r_1/r_2 = 0.5$) as a Function of the Absorption of the Medium

α	ψ_{12}	ψ_{22}	Ψ_{12}	Ψ_{22}	$\mathfrak{A}_1(V)$	$\mathfrak{A}_2(v)$	$\theta_{res,1}$	θ_2
0,1	0,922	0,342	1,560	0,693	0,423	0,375	0,089	0,014
0,5	0,690	0,078	0,789	0,196	0,782	0,685	0,499	0,423
1,0	0,492	0,016	0,512	0,061	0,899	0,795	0,635	0,560

isothermal absorbing medium with temperature T_3. It is assumed that the temperature T_3 of the medium is higher than the temperature T_1 of the inner cylinder ($T_3 > T_1$), and that the surface of the surrounding cylinder is nonadiabatic ($\theta_{res,2} \neq 0$). It is required to determine the density of the hemispherical resulting radiation $\theta_{res,1}$ of the inner cylinder and the temperature T_2 of the surrounding cylinder.

The solution of this problem is given by the following nondimensional computational expressions [4]:

$$\theta_{res,1} = \frac{E_{res,1}}{E_{31}} = A_1\left[\mathfrak{A}_1(V) - \theta_{res,2}\Psi_{12}\right], \tag{15}$$

$$\theta_2 = \frac{T_2^4 - T_1^4}{T_3^4 - T_1^4} = \mathfrak{A}_2(V) - \left(\frac{1}{A_2} + \Psi_{22}\right)\theta_{res,2}, \tag{16}$$

where $\mathfrak{A}_i(V)$ is the resolving absorptivity of the medium ($i = 1, 2$),

$$\theta_{res,2} = \frac{E_{res,2}}{E_{31}} = \frac{E_{res,2}}{\sigma_0(T_3^4 - T_1^4)}.$$

The surface of the inner cylinder is not concave and so $\psi_{11} = 0$, $A_1' = A_1$, $R_1' = R_1$, $\psi_{12}' = \psi_{12}$ and the computational expressions for $\mathfrak{A}_i(V)$ and Ψ_{12}, Ψ_{22}, obtained in [4] can be simplified:

$$\Psi_{12} = \frac{\gamma_2\psi_{12}}{1 - R_1\psi_{12}\psi_{21}'}, \quad \Psi_{22} = \frac{\gamma_2(\psi_{22} + R_1\psi_{12}\psi_{21}')}{1 - R_1\psi_{12}\psi_{21}'}, \tag{17}$$

$$\mathfrak{A}_1(V) = 1 - \frac{A_1\psi_{12}\psi_{21}'}{1 - R_1\psi_{12}\psi_{21}'}, \quad \mathfrak{A}_2(V) = 1 - \frac{A_1(\psi_{21} + \psi_{21}'\psi_{22})}{1 - R_1\psi_{12}\psi_{21}'}, \tag{18}$$

where $\gamma_2 = 1/(1 - \psi_{22})$; $\psi_{21}' = \psi_{21}/(1 - \psi_{22})$.

The generalized average angular radiation coefficients are determined by the approximate method of Mikk [5, 6]. The generalized average and resolving angular radiation coefficients (for $r_1/r_2 = 0.5$), and the resolving absorptivity of the medium (for $A_1 = 0.8$) were calculated numerically for various absorptions of the medium ($\alpha = 0.1,\ 0.5,\ 1.0$).

If $\theta_{res,1} > 0$, then $\mathfrak{A}_1(V) - \Psi_{12}\theta_{res,2} > 0$ implies $\theta_{res,2} < \mathfrak{A}_1(V)/\Psi_{12}$, from which the values of $\theta_{res,2}$ are to be chosen since otherwise the inner cylinder becomes a radiation source.

The results of the numerical calculations of the radiation characteristics (to calculate $\theta_{res,1}$ and θ_2 we took $\theta_{res,2} = 0.2$, $A_2 = 0.9$) for various values of the coefficient of volume absorption ($\alpha = 0.1,\ 0.5,\ 1.0$) are given in Table 1 from which it follows that as α increases the resolving absorptivity $\mathfrak{A}_i(V)$ of the volume V of the medium increases. If the difference between the temperature of the medium T_3 and that of the inner cylinder T_1 is constant an increase in the absorption α of the medium is accompanied by an increase in the temperature T_2 of the radiator (the surrounding cylinder) and hence by an increase in the density of the hemispherical resulting radiation $\theta_{res,1}$ of the inner cylinder.

NOTATION

A_i	is the average coefficient of absorption (degree of blackness) of the surface F_i;
σ_0	is the Stefan–Boltzmann constant;
α	is the coefficient of volume absorption of the medium;
$\mathfrak{A}(M_i, V)$, $\mathfrak{A}_i(V)$	are the resolving local and average absorptivity of the medium;
$\psi(M_i, F_k)$, $\Psi(M_i, F_k)$	are the generalized local geometric and resolving angular radiation coefficients of the elementary surface dF_i at the point M_i on the surface F_k;

ψ_{ik}, Ψ_{ik} are the generalized average geometrical and resolving angular radiation coefficients of the surface F_i on the surface F_k;

$\theta_{res}(M_i)$, $\theta_{res,i}$ are the nondimensional local and average densities of the resulting radiation;

$\theta(M_2)$, θ_2 are the nondimensional local and average temperature factors.

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